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A note on static and dynamic calibration of constant-temperature hot-wire probes

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This note describes a comparative investigation of static and dynamic calibration procedures for standard hot-wire probes. It is demonstrated that nearly the same sensitivity dE/dV can be obtained by both procedures. The discrepancy reported by Perry & Morrison (1971) is shown to be due mainly to a poor approximation of static calibration data over a large velocity range by a constant-exponent power-law function.

1. Introduction

The sensitivity of a hot-wire anemometer probe, defined as the change in output voltage per unit change in fluid velocity, is a function of the magnitude and direction of the velocity, the properties of the fluid and the wire material, the probe geometry, and the electronics used to operate the wire.

The two most common ways of finding the sensitivity are either by plotting the voltage against fluid velocity and differentiating a fitted curve either graphically or numerically ('static' calibration) or by shaking the probe and deducing the sensitivity directly as the ratio of voltage amplitude to shaking velocity amplitude ('dynamic' calibration). Physical reasoning suggests that the results from 'static' and 'dynamic' calibration of hot wires should be nearly identical if the amplitude and frequency of shaking are small. A small difference between the static and dynamic sensitivity may exist owing to a dynamic prong effect occurring at low frequency, which is being investigated by Perry at the present time. The effect is similar to, but much smaller, than the low frequency response of hot-film probes (Bellhouse & Schultz 1967). For short hot-wire probes operated at low overheat ratio errors of the order of 6 % were observed. For probes with a large wire aspect ratio operated at a high overheat ratio, as in this investigation, the error is usually small (Smits & Perry 1975).

Perry & Morrison (1971) (normal hot wire) and Morrison, Perry & Samuel (1972) (yawed hot wire) reported a considerable difference between the results obtained by static and dynamic calibration of a DISA hot-wire probe. This note demonstrates that a large part of this difference is caused by poor approximation of the static calibration data over a large velocity range by any constant-exponent power law. Different static calibration procedures have been suggested (Bruun 1971*a*, *b*; Davies & Patrick 1972), and it is shown that these procedures give good agreement between static and dynamic calibration of hot-wire probes.

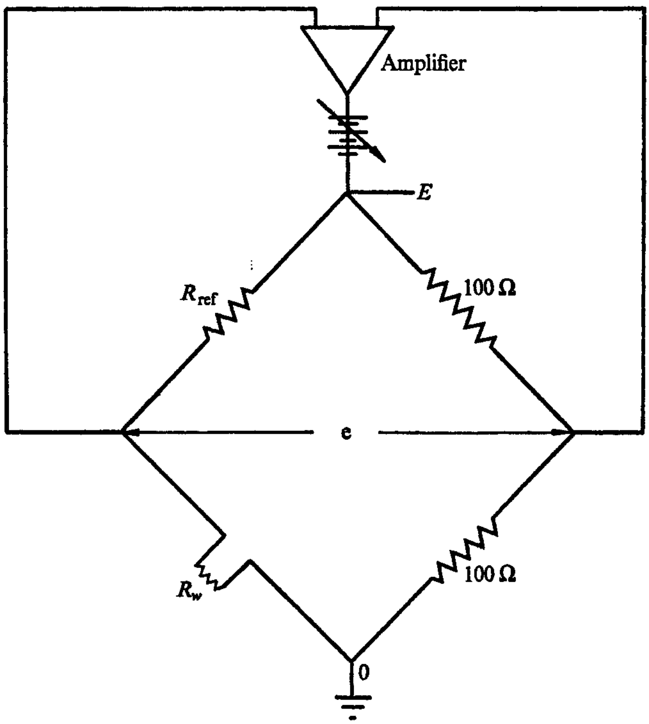

FIGURE 1. Sketch of the ISVR anemometer bridge arrangement.

The anemometer set used in this investigation was of the ISVR type (Davies, Davis & Wold 1967). A diagram of the anemometer bridge arrangement is shown in figure 1. A small variable bias voltage e is maintained in this type of anemometer for optimization of the frequency response.

The sensing element of the hot-wire probes was a 2 mm long 5 μm tungsten wire having a cold resistance R_a (at room temperature) of the order of 7·0–7·5Ω (Bruun 1971*a*). The diameter of the probe support was 3 mm and the support was placed parallel to the mean flow direction. The wire axis was normal to the flow direction. The reference resistance R_{ref} in the ISVR type of anemometer is fixed at 15Ω, giving a hot resistance R_w of the hot-wire probe of approximately twice the cold resistance R_a.

2. Static calibration procedure

Bruun (1971*a*) has demonstrated that the static calibration curves for all hot-wire probes of a given type used with a fixed support orientation and a specified type of anemometer can be approximated with a very high accuracy by the equation

$$E^2 - E_0^2 = Cf(V). \quad (1)$$

In this equation E is the measured voltage output, $f(V)$ a universal shape function for all such probes and C an individual calibration constant for each hot-wire probe. Bruun (1971*a*) also demonstrated that in order to obtain the necessary accuracy with this calibration method it was essential to set E_0 equal to the measured voltage output at zero velocity. It will be demonstrated (§5) that the use of E_0 is not compatible with constant-exponent power-law approximations. The velocity approximation errors introduced by (1) have been shown to be insignificant ($< \pm \frac{1}{2}$ %) for velocities above 10 m/s. At lower velocities the errors were found to increase slightly with decreasing velocity, and be of the order of ± 3 % at 1 m/s.

Bruun (1971*a*) has tabulated a function $f(V)$ corresponding to hot-wire probes having the probe support perpendicular to the mean flow direction. The universal function $f(V)$ corresponding to hot-wire probes having the probe support parallel to the mean flow direction was evaluated using the corrections given by Bruun (1971*a*) and the results are given in table 1. By specifying $f(V)$ in a tabular form errors due to mathematical approximations have been avoided.

From (1) the sensitivity dE/dV for a given hot-wire probe can be evaluated as

$$\frac{dE}{dV} = \frac{C}{2E}\frac{df(V)}{dV}. \tag{2}$$

It is well known (Bradshaw 1971) that it is very difficult to differentiate calibration curves to obtain accurate values of the sensitivity. A different approach, which is described below, was therefore adopted.

The value of C for a given hot-wire probe is determined from (1) by the measurement of E_0 and one accurate measurement of a reference point (V_R, E_R) on the calibration curve. Having determined the value of C the velocity and sensitivity at another calibration point (V, E) are calculated in the following way. From the measured voltage E the corresponding value of $f(V)$ is calculated from (1). The value of the velocity V is then obtained by interpolation of $f(V)$. Both three- and five-point interpolation have been tried, and the results deviate by less than 0·1 %. Three-point interpolations were therefore used in this investigation.

When $d(f(V))/dV$ is known for a given type of hot-wire probe then the sensitivity dE/dV can be obtained from (2) by a similar interpolation of $d(f(V))/dV$. In practice, however, it is more convenient, and just as accurate, to replace the universal function $f(V)$ by

$$f(V) = KV^n, \tag{3}$$

where K and n are functions of the velocity (Bruun 1971*a*).

The values of K and n at each velocity point were evaluated by specifying (*a*) that all calibration points (V, E) should satisfy (3) and (*b*) that the sensitivity calculated from (3) should be identical to the sensitivity given by (2). The derivation of $n(V)$ is described by Bruun (1971*a*), and the values corresponding to hot-wire probes having the probe support parallel to the mean flow direction are given in table 1.

For small velocity fluctuations about a given point on the calibration curve, the values of K and n can be assumed constant, giving the following equation for the sensitivity:

$$dE/dV = \tfrac{1}{2}n(E^2 - E_0^2)/EV. \tag{4}$$

The values of dE/dV corresponding to the universal function with $C = 1$ are given in table 1.

Velocity V (m/s)	Bridge output E (V)	Exponent n	Sensitivity dE/dV (mV(m/s)$^{-1}$)
0	1·167	—	—
0·2	1·268	0·83	404
0·4	1·335	0·717	282
0·6	1·385	0·640	214
0·8	1·423	0·586	170
1	1·454	0·544	141
1·5	1·516	0·532	109·5
2	1·565	0·516	89·7
2·5	1·606	0·510	77·3
3	1·643	0·508	68·9
3·5	1·675	0·505	62·2
4	1·705	0·503	57·0
5	1·758	0·499	49·0
6	1·804	0·497	43·4
7	1·845	0·494	39·0
8	1·882	0·492	35·6
9	1·917	0·490	32·8
10	1·948	0·487	30·4
12	2·005	0·484	26·7
14	2·055	0·480	23·9
16	2·101	0·477	21·7
18	2·142	0·474	19·8
20	2·180	0·471	18·3
25	2·264	0·464	15·4
30	2·336	0·458	13·4
35	2·400	0·451	11·8
40	2·456	0·445	10·6
50	2·550	0·433	8·75
60	2·631	0·420	7·40
70	2·700	0·410	6·45
80	2·761	0·401	5·65
90	2·814	0·390	5·05
100	2·862	0·383	4·55
110	2·906	0·376	4·15
120	2·946	0·370	3·85
130	2·982	0·365	3·55
140	3·017	0·361	3·30
150	3·049	0·359	3·10

Table 1. Universal function $f(V)$ for a 2 mm ISVR hot-wire probe with parallel support orientation

At a given calibration point (V, E) the value of the exponent n was obtained by a three-point interpolation of $n(V)$. Having measured E and E_0 and evaluated V, the sensitivity was calculated from (4).

The uncertainty in the slope evaluation was found to be 2 or 3 times greater than the corresponding velocity uncertainty, giving an accuracy of the sensitivity dE/dV of $\pm$ 1·0–1·5 % under test flow conditions.

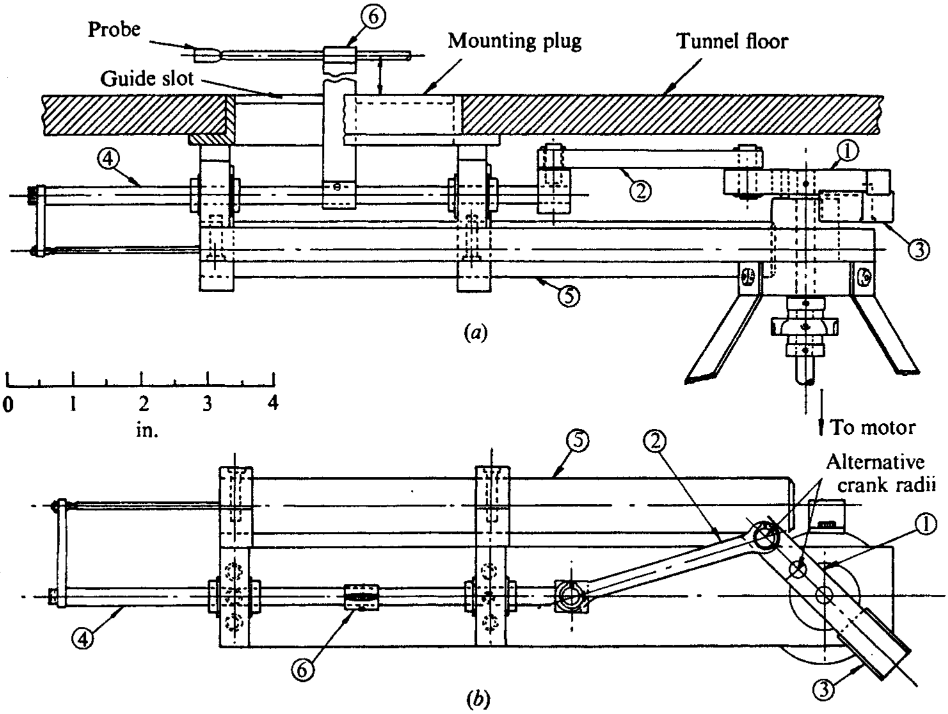

FIGURE 2. Hot-wire dynamic calibrator. Material steel, unless otherwise stated. Counterweight chosen to compromise between lateral and longitudinal vibration. (*a*) Side view. (*b*) Plan view (mounting plug and probe omitted). 1, crank; 2, connecting rod (aluminium); 3, counterweight; 4, sliding shaft; 5, linear displacement transducer; 6, probe support holder (aluminium).

3. Dynamic calibrator

The dynamic calibrations were carried out with the dynamic calibrator at the Department of Aeronautics, Imperial College of Science and Technology, London. The experimental set-up is sketched in figure 2; the calibrator was mounted on the working section of a low-speed open-return wind tunnel (Bradshaw 1972). The mean velocity at the hot-wire position was evaluated from measurements of the static and total pressure with a Betz manometer. The temperature of the air flow was monitored, and by allowing the wind tunnel time to warm up, the temperature could be kept constant during each experiment.

The longitudinal oscillation of the hot-wire probe was provided by the crank mechanism shown in figure 2. A 900 r.p.m. a.c. commutator motor, giving a frequency of approximately 15 Hz, was used to drive the calibrator. As the crank ratio b/a has a finite value of 5·19 a small deviation from a pure sine wave is introduced. Using the notation of figure 3 with $\theta = \omega t$, the length l can be written as

$$l = a\cos\theta + (b^2 - a^2\sin^2\theta)^{\frac{1}{2}}$$

and correspondingly the instantaneous velocity u of the dynamic calibration is

$$u = \frac{dl}{dt} = \frac{dl}{d\theta}\frac{d\theta}{dt} = -\omega a \sin\theta\left[1 + \frac{\cos\theta}{(b^2/a^2 - \sin^2\theta)^{\frac{1}{2}}}\right]. \tag{5}$$

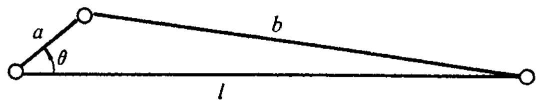

FIGURE 3. Notation for dynamic calibrator. a, crank, length = 14·68 mm; b, connecting rod, length = 76·2 mm.

The r.m.s. value u' of u from (5) is 0·714 ωa as compared with 0·707 ωa for a pure sine wave with the same stroke. The deviation is negligible, and as the distortion is reflected both in u' and in the corresponding r.m.s. value e' of the nonlinear bridge voltage, the sensitivity dE/dV can be evaluated as e'/u'.

An alternative technique is to filter the electrical output so that only the fundamental (corresponding to a sinusoidal oscillation with amplitude a) is passed to the r.m.s. meter.

A linear displacement transducer (Electro Mechanism Ltd, Slough) was connected to the dynamic calibrator, and the output was passed to a RACAL universal timer–counter to obtain the frequency ω necessary for the evaluation of u'. The variation in ω at each experimental point was found to be less than $\pm 0{\cdot}2\,\%$.

The bridge voltage output was passed through a low-pass filter ($f_c = 45$ Hz) to eliminate high frequency vibrations and turbulence. The r.m.s. value e' of the bridge voltage was measured with a DISA 55D35 r.m.s. meter using an integration time of 30 s and a Solation digital voltmeter for the read-out. Both instruments were calibrated to an accuracy of $\pm 1\,\%$.

As the shaking frequency and amplitude were small, error caused by departures from quasi-steadiness of the flow around the probe and support can be ignored, and 'strain gauge' fluctuations in the wire output below the cut-off frequency of the low-pass filter are unlikely as the peak acceleration is only of the order of 10 g. The overall accuracy of the dynamically evaluated sensitivity of e'/u' was therefore of the order of $\pm 1{\cdot}0$–$1{\cdot}5\,\%$.

4. Experimental results

A combined static and dynamic calibration was performed for each wire tested. First the voltage E_0 at zero velocity was measured. Then at each calibration point the following measuring procedure was carried out: (i) the mean velocity V_p was evaluated from a total/static pressure measurement with the Betz manometer; (ii) the mean (static) voltage E was recorded; (iii) a dynamic calibration of the sensitivity was performed; (iv) the mean voltage E was measured again to ensure that no changes had occurred during the dynamic calibration. Four different hot-wire probes were tested for both increasing and decreasing velocity and similar results were obtained in both cases. The velocity range covered was 10–38 m/s.

The static mean voltage E was plotted as a function of the velocity V_p evaluated from the total/static pressure measurements. The static calibration data must, apart from small experimental errors, fall on a smooth curve. If large jumps or kinks occurred in the static calibration data, then changes in the calibration

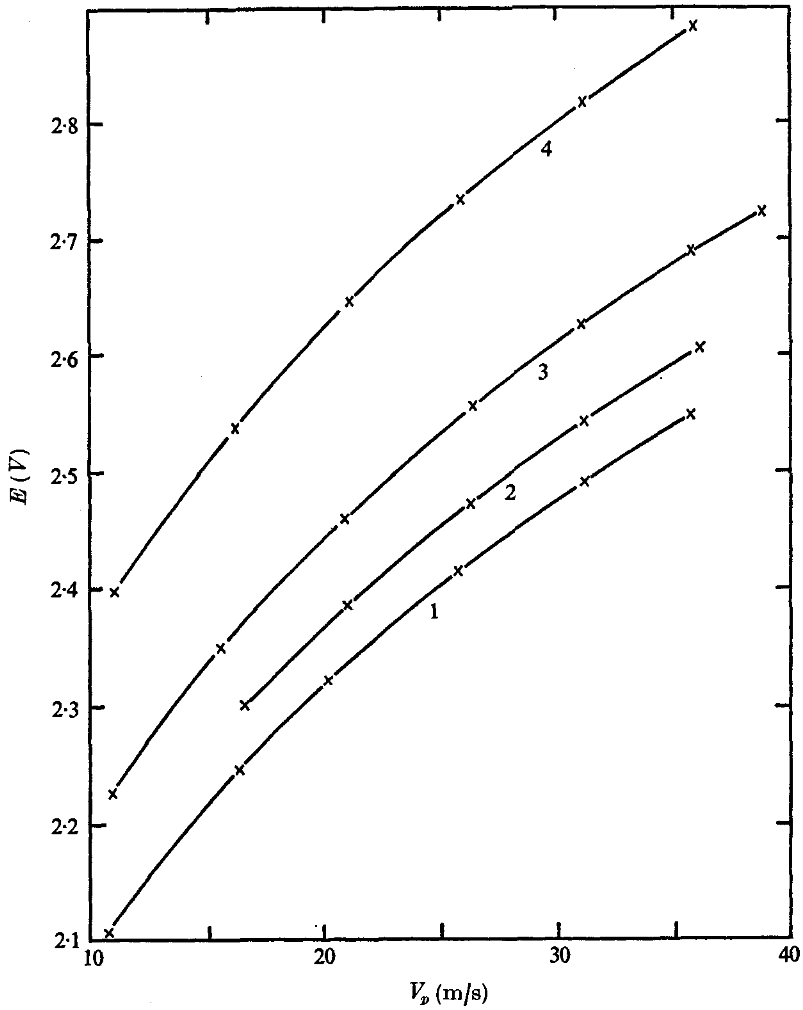

FIGURE 4. Static calibration curves for four hot-wire probes. Wire 1, $E_0 = 1{\cdot}323$, $C = 1{\cdot}071$; wire 2, $E_0 = 1{\cdot}354$, $C = 1{\cdot}108$; wire 3, $E_0 = 1{\cdot}386$, $C = 1{\cdot}194$; wire 4, $E_0 = 1{\cdot}503$, $C = 1{\cdot}362$.

characteristics were assumed to have occurred during the experiment, and the results were discarded. A static calibration curve for each of the four hot-wire probes tested is shown in figure 4. Optimum conditions for comparison of static and dynamic calibration are seen to exist in all four cases.

For each test the constant C in the static calibration [equation (1)] was evaluated from the calibration point with maximum velocity (V_{max}, E_{max}). At each calibration point a velocity V_p was calculated from the total/static pressure reading, and from the measured static voltage E a static velocity V_s and sensitivity dE/dV_s were evaluated as described in §2. The corresponding dynamic sensitivity dE/dV_d was obtained as outlined in §3.

To demonstrate the accuracy of the static velocity calculation and the difference between the static and the dynamic evaluation of the sensitivity, the

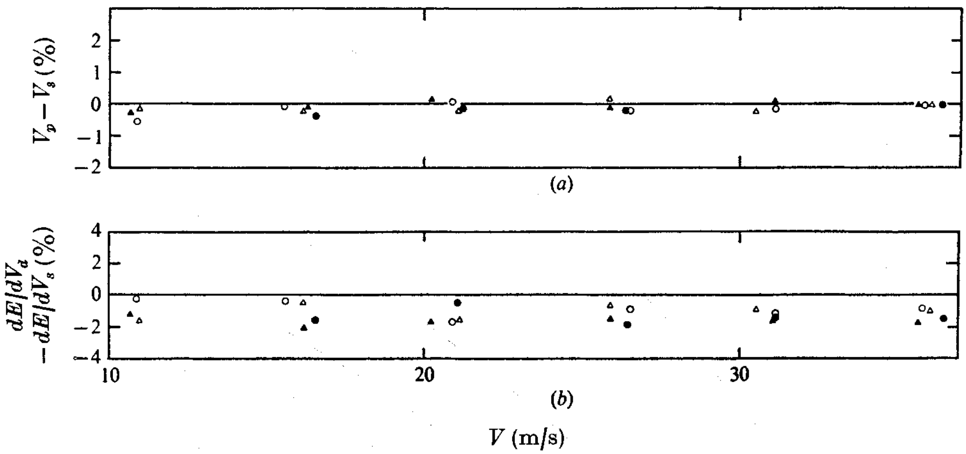

FIGURE 5. Comparison of static and dynamic calibration. (a) Difference in velocities evaluated by total/static pressure measurement and static calibration. (b) Difference in sensitivities evaluated by dynamic and static method. ▲, wire 1; ●, wire 2; ○, wire 3; Δ, wire 4.

differences $V_p - V_s$ (%) and $dE/dV_d - dE/dV_s$ (%) have been plotted in figure 5 for the four hot-wire probes tested. The velocity V_s evaluated by the static calibration method is seen to agree to within ±0·5 % with the velocity V_p evaluated in §2. The difference between the sensitivities obtained by the static and the dynamic calibration method is seen to vary between zero and −2 %. The accuracies of the two methods are both of the order of ±1·0–1·5 %. It is therefore possible within the experimental accuracy to obtain the correct value of the sensitivity dE/dV by both static and dynamic calibration of hot-wire probes.

In many practical hot-wire applications, the above accurate static calibration method cannot be justified, owing to the computational procedures required. Approximate analytical equations have therefore been developed to represent static calibration data. The accuracy of such methods is discussed in §5.

5. Approximate analytical calibration laws

The universal function $f(V)$ has been shown in §4 to be an accurate representation of static calibration data. The corresponding static sensitivity function $dE(V)/dV$ (table 1) has been plotted in figure 6.

To demonstrate the inadequate approximation of static calibration data over a large velocity range by a constant-exponent power law, a least-squares-fit method was applied to the calibration data (table 1) in the velocity range 10–30 m/s using values of n = 0·40, 0·45 and 0·50. The corresponding sensitivity curves are plotted in figure 6. The least-square errors ϵ^2 corresponding to n = 0·40, 0·45 and 0·50 were 0·000016, 0·000096 and 0·00053. Further examination of the exponent range 0·40–0·45 revealed that a minimum value of ϵ^2 of 0·0000041 was obtained for n = 0·41. The power law $V^{0·41}$ is therefore the most accurate power-

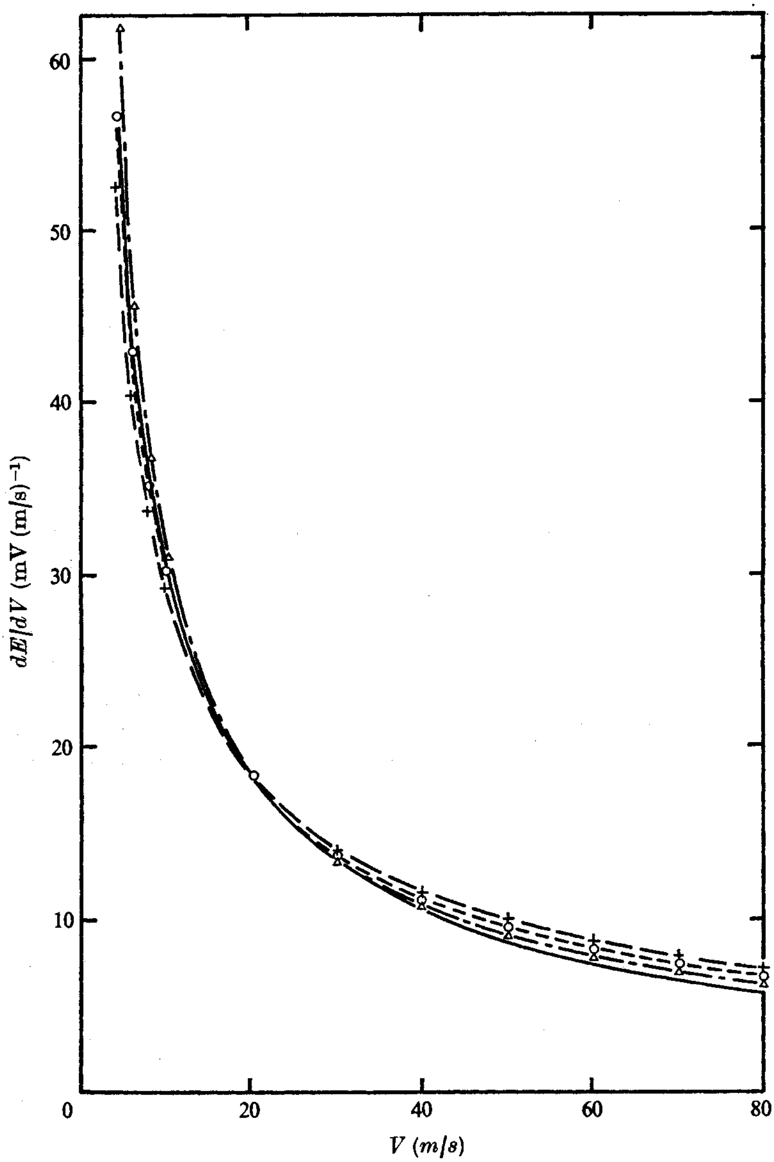

FIGURE 6. Comparison of static sensitivity (table 1), solid curve, and three constant-exponent power-law approximations. Δ—·—Δ, $n = 0{\cdot}40$; ○ - - - ○, $n = 0{\cdot}45$; + —— +, $n = 0{\cdot}50$.

law approximation in the velocity range 10–30 m/s. The sensitivity plots show a similar result.

At high velocity all three power laws are seen to be poor approximations. Below 10 m/s the power law $V^{0{\cdot}40}$ gives values for the sensitivity which are too large, while values which are too low are obtained using the power law $V^{0{\cdot}50}$. The power law $V^{0{\cdot}45}$, however, provides a good fit in this region, a result which is in agreement with the low velocity results of Collis & Williams (1959). The intersection value A in the power-law approximation was in this case evaluated as approximately $0 \cdot 9E_0^2$, and Bruun (1971*b*) has shown that accurate results can be obtained with the universal-function method only if A is set equal to E_0^2. An individual calibration of each hot-wire probe is therefore necessary if a constant-exponent power-law approach is used.

A similar power-law comparison was carried out by Perry & Morrison (1971). The general trend of their power-law approximations is as in figure 6. The sensitivity was evaluated by graphical differentiation giving a different vertical separation of the sensitivity curves corresponding to $n = 0 \cdot 40$, $0 \cdot 45$ and $0 \cdot 50$.

These evaluations have clearly demonstrated that no constant-exponent power law will give a good approximation of static calibration data over a large velocity range. A more complex mathematical function is needed for this purpose. Bruun (1971*b*) and Davies & Patrick (1972) have demonstrated that the function

$$E^2 = A + BV^{\frac{1}{2}} + CV \tag{6}$$

provides a good approximation to the static calibration data over a large velocity range. The value of the constant C was found to be approximately $-0 \cdot 015$. The term CV can therefore be interpreted as a correction to the usual constant-exponent power-law approximation, which explains the good approximation of (6) to the static calibration data. In the investigation by Bruun (1971*b*), the constant A was set equal to E_0^2, which permits the concept of a universal function to be used. The value of A ($\sim 0 \cdot 9E_0^2$) used by Davies & Patrick (1972) was obtained by a least-squares-fit method, applied to the measured calibration data for a given hot-wire probe. The accuracies of the sensitivities evaluated by the two methods were both found to be of the order $\pm$ 1–2 % in the whole of the velocity range 5–60 m/s.

6. Conclusion

This note has demonstrated that provided static calibration is performed with the high accuracy necessary then the values of the sensitivities dE/dV of a hot-wire anemometer obtained by static and dynamic calibration procedures respectively agree to within the experimental accuracy. Constant-exponent power laws have been shown to introduce serious approximation errors when applied over a large velocity range. Good agreement can, however, be obtained when an extended power law [equation (6)] is used.

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